Search and the dynamics of house prices and construction

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Abstract

The dynamics of house prices, vacancies, and construction are studied in a search model of the housing market in which both construction and the entry of buyers are determined in equilibrium. The theory accounts for substantial positive serial correlation in house price appreciation in the short/medium run with mean reversion in the long run, even if housing dividends are strictly mean–reverting. Quantitatively, however, the theory understates the magnitude of fluctuations in both prices and population growth. Competitive search contributes to volatility as flutuations in buyers' share of the surplus in housing transactions increase the short run response of house prices to fluctuations in local income.

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1 Introduction

In this article we explore the consequences of search and matching frictions in housing markets for price and construction dynamics. We consider an environment in which both the entry of new buyers and the construction of new houses are determined endogenously in equilibrium. We show that in the presence of search frictions, house prices may exhibit short term momentum, even if housing dividends are strictly mean–reverting. In the long run, the construction of new homes eventually reduces the ratio of buyers to houses for sale, and prices converge to their long-run values. In a calibrated version of the basic model with housing dividends represented by local incomes, we find that substantial price momentum can arise in the short-term. Although this basic model cannot account for sufficient variance in house prices relative to the magnitude of income fluctuations, we consider several generalizations of the basic model that can be expected to improve its performance.

Housing market dynamics in US cities can be characterized by several key stylized facts, which we discuss in Section 2.[1] Firstly, most time-series variation in house prices is local in nature, not national.[2] This has motivated researchers to use local factors such as income, regulations and construction costs to account for price movements. Secondly, houses prices are very volatile when compared with per capita incomes and rents. This appears to be true both at the in relative terms across cities and in aggregate. A third key observation is that there is strong positive serial autocorrelation in house price appreciation over the short term, but mean reversion in prices over longer periods.[3] Finally, as Glaeser and Gyourko (2006) highlight, there is strong short run persistence of construction rates with long–run weak mean-reversion, and high volatility of construction levels within markets.

While the movements in house prices are reasonably well documented, Capozza, Hendershott and Mack (2004) point out that a well–developed behavioural theory to account for them has proved difficult to construct. Since the work of Case and Shiller (1989) and Cutler, Poterba and Summers (1991), it has been recognized that movements in housing prices (like those of many other assets) pose a challenge to strict versions of the efficient markets view. In particular, the fact that the strong positive autocorrelation of house price appreciation does not appear to be explained by fundamentals suggests that a simple asset–pricing approach

[1] Glaeser and Gyourko (2008) also document these facts, but our approach is somewhat different.

[2] This has been noted in the US by Abraham and Hendershott (1996) and Del Negro and Otrok (2006), but also in Canada by Allen, Amano, Byrne and Gregory (2007).

[3] See Abraham and Hendershott (1996), Capozza, Mack and Mayer (1997), Malkpezzi (1999) and Meen (2002).

alone may be of limited value.[4] Many authors have gone further to argue that to explain housing market dynamics, one must introduce aspects of irrationality and/or rule–of–thumb behaviour.

There are several good reasons to suspect that search and matching frictions may play an important role in housing markets. For example, the observed positive comovement of prices and sales (Rios-Rull and Sanchez Marcos, 2007). and the fact that prices and sales are negatively correlated with average time on the market (Krainer, 2008). As first noted by Peach (1983) and more recently documented by Caplin and Leahy (2008), there is significant negative correlation between vacancies and price appreciation. Diaz and Jerez (2009) suggest that allowing for competitive as opposed to random search can affect price movements because the division of the surplus between buyers and sellers may also depend on the tightness of the market.

In this paper, we develop a framework that introduces frictions of these types into a housing market where both the entry of new buyers and the construction of new houses evolve endogenously. The value of living in a particular city is determined by an exogenous housing dividend which we think of as relative income. New buyers enter the market as renters and search for a house whenever the expected value of doing so exceeds their next best alternative. New houses are constructed and offered for sale or for rent whenever the expected value of doing so is high enough relative to the costs of construction. Existing owners may also put their houses up for sale or rent them out and exit the market when they experience changes in their life situation, which we model as the realization of an exogenous shock. We model trade in the housing market as characterized by directed search as proposed by Moen (1997). We establish the existence of a unique stationary growth path characterized by constant population growth (of home-owners and renters) and construction rates.

We study the implications of shocks to relative income in two calibrated versions of the model. We begin with a version in which the matching function is Cobb-Douglas. This implies that surplus shares are independent of market tightness, so that the model is effectively equivalent to a random search model with Nash bargaining. We find that this model can generate short term price momentum even in the absence of persistent income growth (ie. even if shocks to incomes follow an AR(1) process). The reason is that, although an initial rise in the value of living in a city generates an immediate increase in search activity, it takes time for potential buyers to match with a house (indeed, the likelihood that an

[4] Case and Shiller (1989) argue that serial correlation in rents does not explain momentum in price changes.

individual buyer finds a match may decline somewhat). Although, sales and the probability of selling rise immediately, construction of new housing takes time to respond. Even if the value of searching subsequently declines (due to mean reversion), the number of searchers may continue to rise and the stock of vacancies declines. Consequently, the ratio of buyers to sellers continues to rise in the short term, further driving up the rate at which houses sell and hence the value of a vacant house. Since, in equilibrium, house prices partly reflect this value, they rise too. Eventually, the stock of potential buyers starts to fall as they are absorbed into the owner-occupied housing market, and the decline in vacancies slows (and eventually reverses) as construction rates catch up. This causes the ratio of buyers to sellers to decline so that prices eventually start to revert back towards their steady state values.

While this model generates both momentum and mean-reversion in prices, which models without search frictions typically cannot, it accounts for less variance in both prices and construction rates than, for example, the theory of Glaeser and Gyourko (2006). When, however, we assume a generalized "urn-ball" variety as studied byAlbrecht, Gauthier, and Vroman (2003), the variance of house prices rises significantly. In such an environment, the share of the surplus received by the buyer in any housing transaction is a decreasing function of the ratio of buyers to sellers. Consequently, as buyers enter, the house price increases by more than it would in a model with a fixed surplus splitin the random search case as buyers enter, then falls by more as more houses are constructed in order to catch up with demand. Thus, the introduction of competitive search increases the overall variance of prices, while maintaining the initial momentum in growth rates.

Our analysis is closely related to two other recent papers on housing markets. Diaz and Jerez (2009) also develop and calibrate a competitive search model of the housing market and compare the implications of alternative matching functions. They adopt a version of Wheaton's (1990) model with no entry and no house construction and their conclusions are largely based on a steady state analysis. Our analysis goes beyond this, but is motivated in part by their insight that competitive search may magnify the effects of exogenous changes on house prices. Glaeser and Gyourko (2006) develop a dynamic, rational expectations model with no search frictions. House prices are determined by relative income movements, which induce entry in the short run, and housing supply conditions which pin down prices in the long run. They calibrate a version of their model and study its dynamics driven by an estimated ARMA(1,1) process on incomes. The possibility of short–term price momentum and mean reversion in prices and construction arises because of the hump-shaped pattern of relative incomes.[5] They find that their model is reasonably successful in accounting for longer term movements in prices and construction and for overall volatility in the median market. However, their calibrated model cannot generate any short term momentum in prices and persistence in construction rates is too low.

Although a number of other papers have studied the role of search and matching frictions in housing markets (e.g. Wheaton (1990), Krainer (2001) Albrecht at al. (2007), Head and Lloyd-Ellis (2008)), these generally focus on the market for existing housing treating the housing stock as fixed and consider steady state implications. While Caplin and Leahy (1998) do consider the non-steady state implications of their model, they also assume a fixed housing stock. In contrast, we focus on the role of frictions for the transitional dynamics of prices and construction of new homes. Although we do allow for turnover of existing homes, this is not crucial for the qualitative nature of price and investment dynamics (although it does matter quantitatively). Models of housing investment and construction (e.g. Davis and Heathcote, 2007 and Glaeser and Gyourko, 2006), on the other hand, generally abstract from trading and matching frictions in the market for houses in order to focus on supply side factors. In this paper we bring together aspects of both literatures.

In Section 2 we document some of the key empirical features of housing market dynamics. We then go in Section 3 to develop the basic strucure of our model of housing and in section 4 we characterize the equilibrium. Section 5 discusses the stationary growth path of the economy and Section 6 discusses some extensions to the basic model. Section 7 develops a calibration scheme and in Section 8 we study the qualitative dynamic implications of the model. In Section 9 we evaluate how well the model does quantitatively and Section 10 offers some concluding remarks. All proofs and extended derivations are in the appendix.

2 Empirical properties of house prices, incomes, and population

2.1 Univariate Characterization of Prices

In this section we characterize the dynamics of *relative* house prices for a typical large US city. To do this we use a panel of 101 cities with annual data between 1977 and 2008. We

[5] The also assume utility is decreasing in local population size which has a dampening effect on prices. However, in their calibration this effect is tiny so, in fact, the shock process drives everything.

estimate the following fixed effects regresssion

$$\Delta P_{ct} = \sum_{i=1}^{T} \alpha_i \Delta P_{ct-i} + \beta P_{ct-1} + \mu_t + \mu_c + \varepsilon_{ct}$$

where $\Delta P_{ct} = P_{ct} - P_{ct-1}$ and P_{ct} represents the log of the average house price in city c and time t. The terms μ_c and μ_t represent city level and time fixed effects, respectively, and T is the number of lags. The first column of Table 1 shows the estimates for the full sample. We found that 1 lags of the growth in prices was necessary and sufficient to describe the evolution of prices.

Table 1: Annual Panel (fixed effect) estimates for price process

Parameter	Full Sample (1980-2008) 101 cities	Coastal Cities (1980-2008) 30 cities	Inland Cities (1980-2008) 71 cities	Truncated Sample (1980-2000) 101 cities
α_p	0.66 (41.97)	0.66 (23.57)	0.57	0.51 (27.64)
β_p	-0.14 (24.17)	-0.15 (13.44)	-0.14	-0.15 (18.35)
City dummies	Yes	Yes	Yes	Yes
Time dummies	Yes	Yes	Yes	Yes
σ_ε	0.039	0.038	0.037	0.038
σ_μ	0.010	0.01	0.007	0.009
ρ	0.069	0.062	0.037	0.053
R^2 within	0.61	0.79	0.49	0.42
R^2 between	0.35	0.38	0.52	0.59
R^2 overall	0.60	0.78	0.49	0.42
# of obs.	2929	870	2059	2121

The IRFs for the implied AR(2) process followed by housing prices for the average city in each sample are illustrated in Figure 1. As can be seen, the house prices dynamics can be characterized as exhibiting a hump-shaped pattern, with initial autocorrelation and subsequent mean-reversion. The peak in the IRF occurs after about 3 years and after about 6 years the autocorrelation becomes negative.

Several authors (e.g. Capozza et al. 2004, Glaeser et al. 2008,) have argued that there is substantial heterogeneity across housing markets. In particular, they argue that coastal and inland markets faces very different constraints which are reflected in housing construction costs and price dynamics. To assess the implications of this we divide our sample into coastal and inland cities and re-estimate the panel regression (see columns two and three of Table 1). Using these we generate the implied IRFs which are depicted in the bottom panel of Figure 1. While the overall hump-shaped dynamics are similar, one can see that the initial autocorrelation is much more substantial for coastal cities, whereas the prices remain persistently high for inland cities, mean-reverting more slowly.

Recent events in the housing markets have been particularly unusual with average prices rising rapidly until 2007 and then collapsing with the housing bust. To assess the implication of this turmoil, we estimate the panel regression on a truncated sample period going only up to 2000. Note that because we are looking at price movements across cities *relative* to the mean, the average run up in prices between 2000 and 2008 does not necessarily imply that our estimates must be any different. Nonetheless we do find that the impied IRF based on the estimated model during the early period involve less momentum. Interestingly, the estimates and the IRF for the truncated sample is similar to those for inland cities estimated over the full sample period.

2.2 A Structural Panel VECM

The theory that we develop below implies the following: a positive shock to income, which has persistent effects, induces households to enter a city more rapidly which, in turn, drives up the demand for housing relative to trend. The consequent rise in house prices (and rents) stems the rate of entry to some extent. Depending on the extent of frictions in housing markets, the ratio of buyers to sellers may continue to rise, driving up prices further. However, eventually incomes mean revert causing entry to slow relative to construction, prices decline and and the economy coverges back to its long run trend.

This broad description motivates us to consider a structural panel VECM of the following form:

$$\mathbf{\Gamma}\Delta\mathbf{X}_{ct} = \mathbf{A}\Delta\mathbf{X}_{ct-1} + \mathbf{B}\mathbf{X}_{ct-1} + \mathbf{F}_c + \mathbf{F}_t + \boldsymbol{\varepsilon}_{ct}$$

where $\mathbf{X}_{ct} = [Y_{ct}, P_{ct}, N_{ct}]'$ denotes the vector of incomes per capita, average house prices and populations in each city at each date, $\mathbf{\Gamma}$, $\mathbf{A}$ and $\mathbf{B}$ are structural parameter matrices, $\mathbf{F}_c$ is a vector city fixed effects, $\mathbf{F}_t$ is a vector of time fixed effects and $\boldsymbol{\varepsilon}_{ct} = [\varepsilon^Y_{ct}, \varepsilon^P_{ct}, \varepsilon^N_{ct}]'$ are the structural shocks. To estimate the structural parameters of this model, we must impose a set of identifying restrictions. Specifically, we assume that that the structural shocks are orthogonal and that $\mathbf{\Gamma}$ is triangular. The latter assumption effectively imposes the assumption that prices and population do not affect per capita incomes contemporaneously and that

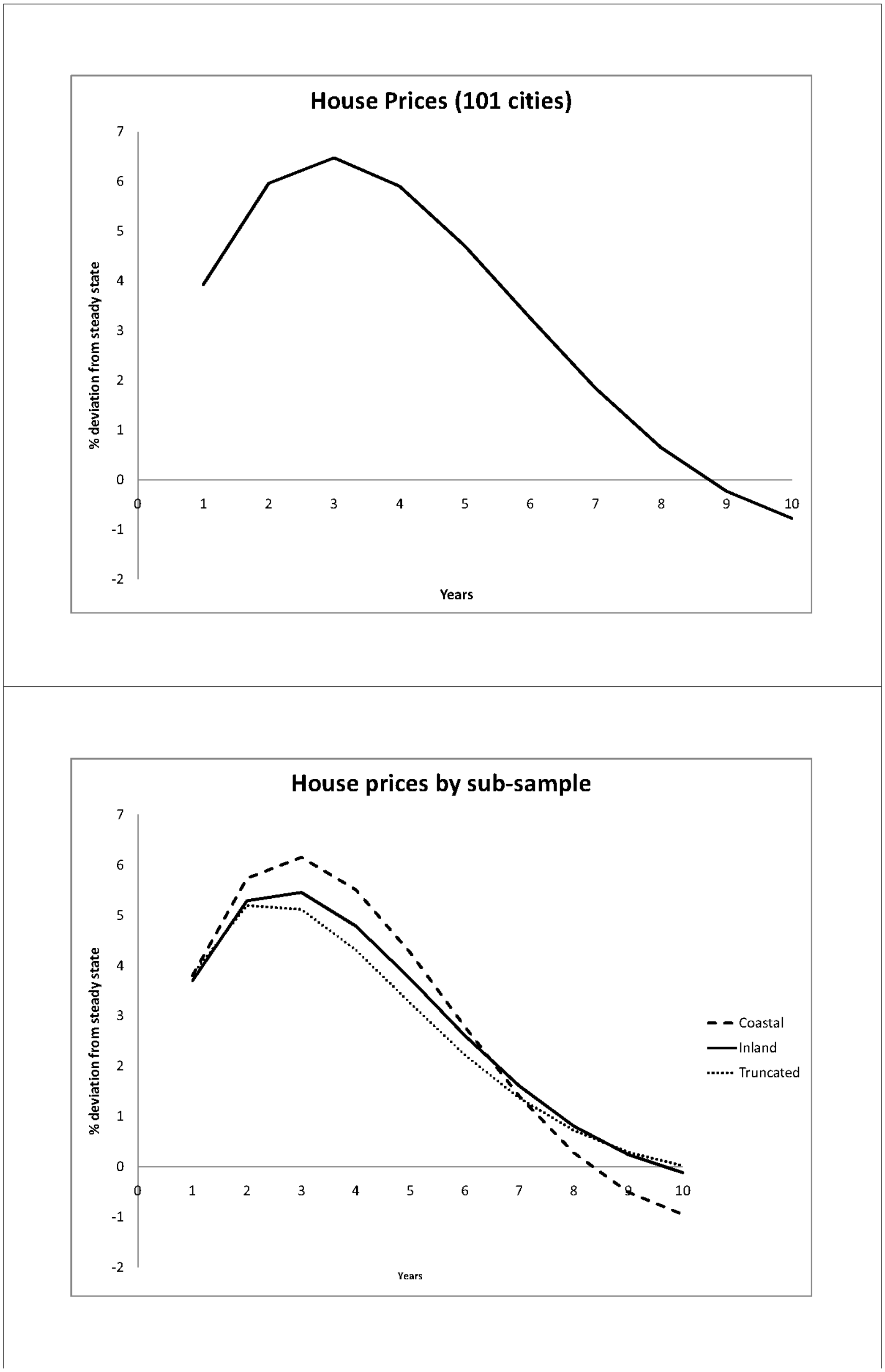


Figure 1: IRFs for annual house prices

prices do not affect population contemporaneously. These restrictions seem reasonable in our case. For example, although there could be agglomeration effects of population on income, this is likely to be a longer term effect. Similarly, the effect of house prices on incomes and populations seems likely to occur only with a lag.[6]

Table 2: Moments from SPVECM – Income Shock

		Relative Std. Dev	Correlation with Income	Autocorrelation in growth rates 1 year	2 year	3 year	4 year
Incomes	Overall	1.0	1.0	0.1300	-0.0509	-0.0792	-0.0750
	Coastal	1.0	1.0	0.1250	-0.0125	-0.0325	-0.0339
	Inland	1.0	1.0	0.1802	-0.0197	-0.0789	-0.0973
	Truncated	1.0	1.0	0.1650	-0.0368	-0.0772	-0.0776
Prices	Overall	1.6086	0.8775	0.7921	0.3872	0.0243	-0.2201
	Coastal	1.0054	0.8362	0.7886	0.3699	0.0139	-0.2159
	Inland	1.7805	0.8753	0.7979	0.4253	0.1170	-0.0924
	Truncated	1.5266	0.9231	0.7794	0.3799	0.0659	-0.1286
Population	Overall	0.4343	0.8453	0.6823	0.1987	0.0819	-0.2144
	Coastal	0.5892	0.8260	0.7537	0.4173	0.1975	0.0807
	Inland	1.0518	0.4270	0.7796	0.4591	0.1973	0.0057
	Truncated	0.7038	0.6216	0.7379	0.4224	0.1864	0.0370

Table 5 provides key moments for incomes, prices and population movements based on the SPVECM in each of the sub-sanples. These moments arise from isolating the impact of structural shocks to income. Several, key observations are apparent. First, the standard deviation in prices is between 1 and 1.8 times higher than that of per capita incomes. This is lower than that suggested by Glaeser and Gyourko (2008) and stems from the fact that the SPVECM isolates the effect of income shocks. Second, the standard deviation of city populations is between 0.4 and 1 times the standard deviation of income. Both prices and populations are strongly positively correlated with per capita incomes, although for inland cities this correlation seems weaker. The high and more persistent autocorrelation in both house price appreciation and population growth relative to income growth can also be observed in all the sub-samples. The degree of price momentum implied here is similar to that estimated by Glaeser and Gyourko (2008) based on a panel regression of price growth on its own lag, but is larger than implied our univariate ECM estimated above. Figure 2 depicts the impulse response functions in repsonse to a one percent income shock for each of the sub-samples.

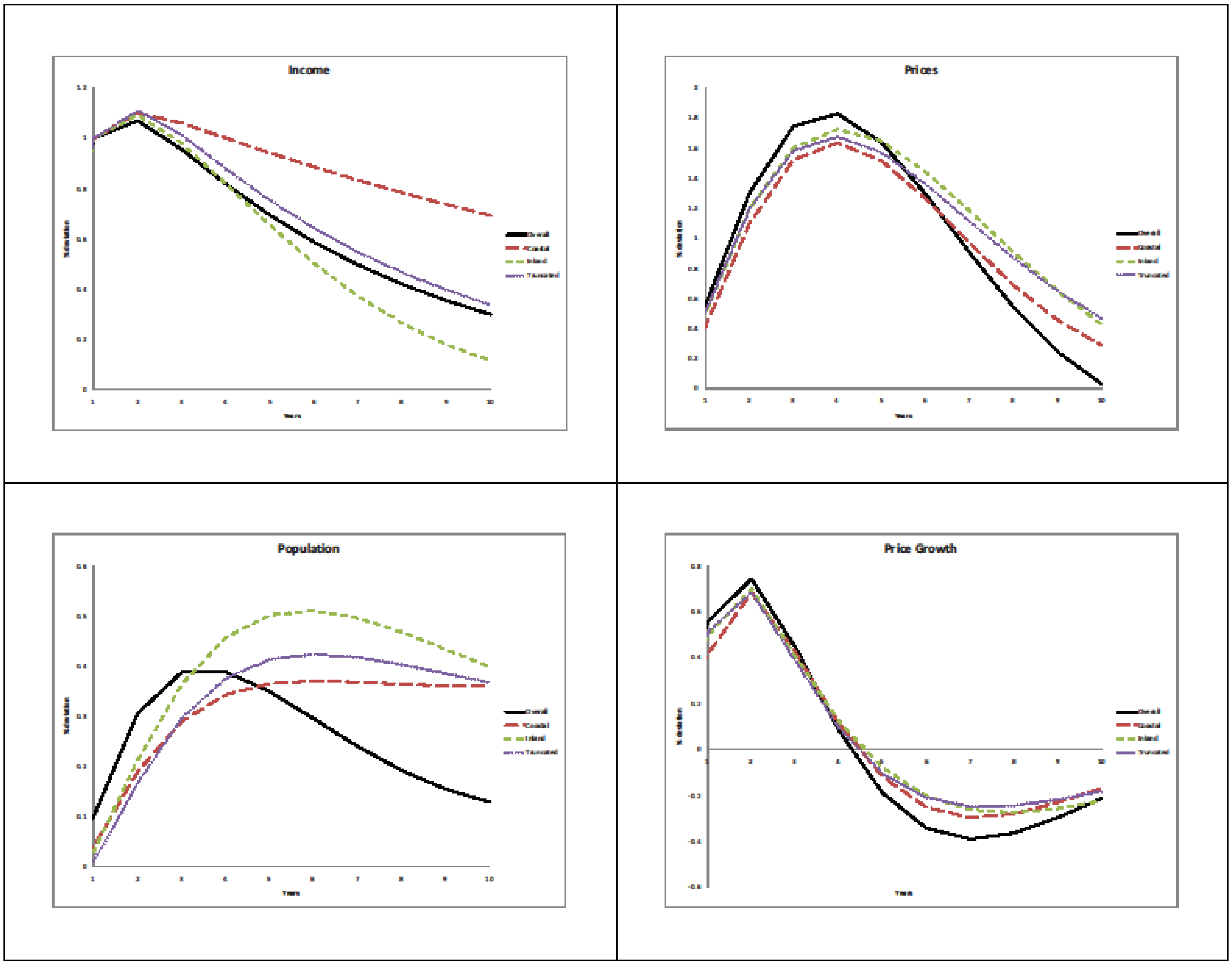


Figure 2: IRFs based on structural VECM

3 The Environment

Time is discrete and indexed by t. We consider activity in a single housing market (*e.g.* a city), treating activity outside the market as exogenous. The total population of the economy, Q_t, including households outside this particular housing market, grows at rate μ. At each date there is a stock of *ex ante* identical housing units H_t which can either be owned or rented. There is also a measure of existing homeowners, N_t and a measure of existing renters. Renters consist of those who are currently searching to buy a house, B_t, and those who plan to remain as renters, F_t. The housing market therefore consists of vacant housing

[6]These restrictions are consistent with our theory anyway.

which is for sale $S_t = H_t - N_t - B_t - F_t$ and the measure of potential buyers, B_t, who are currently renting. Houses for sale include both new houses that are currently owned by developers and houses which are being sold by existing owners who are moving elsewhere. Each period, households enter the city either as perpetual renters or as potential buyers. All agents discount the future at the same rate $\beta \in (0,1)$. We assume that capital markets are perfect and that the interest rate is $1/\beta$.

Households supply two types of labour: general labour, a unit of which is supplied inelastically by each household, and construction labour, l_t, whose supply is endogenous. General labour earns y_t per unit supplied and the construction labour of each household earns w_t per hour. Preferences over consumption, c_t, construction labour supply and housing are given by:

$$u_t^j(c_t, l_t, z_t^j) = c_t - v(l_t) + z_t^j \tag{1}$$

where

$$v(l_t) = \frac{l_t^{1+\frac{1}{\eta}}}{\zeta^{\frac{1}{\eta}}\left(1 + \frac{1}{\eta}\right)} \tag{2}$$

and $z_t^j \in \left\{z^R, z^H\right\}$ denotes housing utility derived from renting or owning, respectively. The quasi-linearity of preferences together with perfect capital markets imply that households are indifferent about the timing of their consumption. Consequently their optimization problem is equivalent to one where consumption is replaced by their income net of any rental payments, r_t

$$c_t = y_t + w_t l_t - r_t. \tag{3}$$

The solution to the (static) household labour supply problem yields

$$l_t = l^s(w_t) = \zeta w_t^\eta. \tag{4}$$

Substitution yields the following effective utilities for owners and renters:

$$\begin{aligned} u_t^H(y_t, w_t) &= y_t + x(w_t) + z^H & (5)\\ u_t^R(y_t, w_t) &= y_t + x(w_t) + z^R - r_t & (6) \end{aligned}$$

where

$$x(w_t) = w_t l^s(w_t) - v(l^s(w_t)) = \frac{\zeta w_t^{1+\eta}}{1+\eta}. \tag{7}$$

We assume that each period any house that is not currently owner-occupied may be offered for sale or rented. A house that is rented earns the rent r_t, but there is also a maintenance fee m that must be paid each period that the house is rented. There are no frictions in the rental market. Houses that are designated for sale must remain vacant for at least one period and the value of a vacant house at time t is denoted V_t. It follows that the value of a house that is not currently owner–occupied is

$$\widetilde{V}_t = \max\left[r_t - m + \beta E_t \widetilde{V}_{t+1}, \quad V_t\right] \tag{8}$$

Existing homeowners may leave the city for exogenous reasons. Specifically, we assume that with probability π owners experience a taste shock that drives their utility to zero if they remain in their current house. In this event they move out immediately, receive an exogenous continuation value, Z, and either put their house up for sale or rent it. We assume that $Z = \bar{u}^R/(1-\beta)$ where $\bar{u}^R$ is the (endogenous) steady-state value of u_t^R. It follows that the present value, J_t, of being a homeowner is given by

$$J_t = u_t^H + \beta\left[\pi\left(Z + E_t\tilde{V}_{t+1}\right) + (1-\pi)E_t J_{t+1}\right]. \tag{9}$$

We impose the boundary condition that $\lim_{T\to\infty}\beta^T E_t J_{t+T} = 0$.

In order to construct a new house, developers must first purchase land at unit price q_t. New houses either for rent or to own are constructed according to a simple linear production function using labour effort, L_t:

$$H_{t+1} - H_t = \phi L_t \tag{10}$$

where ϕ is a productivity parameter. Houses constructed at time t become available either for sale or for rent at time $t+1$ and there is no depreciation. We assume there is free entry into construction. We denote the stock of housing that is for rented, H_t^R, so that the stock that is either owner-occupied or designated for sale is $H_t - H_t^R$.

In our basic model, we assume that the price of land is constant through time:

$$q_t = \bar{q} \tag{11}$$

In reality land prices may vary in response to changes in the value of living in a particular location (see Davis and Heathcote, 2007). For now we keep this constant, but later we consider the implications of endogenizing land prices.

Once developers have built a house, it can either be rented or designated for sale, in which case it will remain vacant for at least one period. Note that, since both new houses and existing houses that are designated for sale yield zero flow utility and since all agents discount the future at the same rate, such houses have the same value to the sellers, V_t.

We assume that the market for housing is characterized by directed search as proposed by Moen (1997). Although all vacant houses are identical, they may, in principal, be sold in a variety of submarkets indexed by i. Each submarket is distinguished by the house sale price, P_t^i, to which sellers must commit if they enter. Let the number of buyer and sellers who enter submarket i be denoted B_t and S_t^i, respectively. The number of matches per period in submarket i is assumed to be determined by the common matching function,

$$M_t = M(B_t^i, S_t^i), \tag{12}$$

where M is increasing both arguments and exhibits constant returns to scale. We let ω_t^i denote the ratio of buyers to sellers in market i, which we refer to as the "tightness" of the market. It follows that a buyer who enters sub-market i will find an appropriate vacant house with probability

$$\lambda_t^i = \frac{M(B_t^i, S_t^i)}{B_t^i} = \lambda(\omega_t^i) \tag{13}$$

where $\lambda'(\cdot) < 0$. Similarly, a seller who offers his house for sale in submarket i will find a buyer with probability

$$\gamma_t^i = \gamma(\omega_t^i) = \frac{M(B_t^i, S_t^i)}{S_t^i} = \omega_t^i \lambda(\omega_t^i) \tag{14}$$

where $\gamma'(\cdot) > 0$.

It is costless to offer a vacant house for sale. It follows that for a seller to enter sub-market i, he must expect to receive at least the value of the vacant house. That is

$$V_t = \max_i \left[\gamma(\omega_t^i) P_t^i + (1 - \gamma(\omega_t^i)) \beta E_t \tilde{V}_{t+1} \right]. \tag{15}$$

We assume that $P_t^i > \beta E_t V_{t+1}$ so that sellers always sell if offered the price. We verify later that this holds in equilibrium. Free entry of sellers into submarket i with price P_t^i will drive $\gamma_t^i = \gamma(\omega_t^i)$ down until (15) holds with equality.

Potential buyers incur zero search costs and each period must decide which submarket to enter, if at all. We assume that renters can observe prices and market tightness in each submarket. If they are matched they may choose to buy the house at price P_t^i in which case they become a homeowner in the next period. Otherwise they remain unmatched and continue to search. The present value of being a renter, W_t, is therefore given by

$$W_t = u_t^R + \max_i \left\{ \lambda(\omega^i) \left(\beta E_t J_{t+1} - P^i \right) + (1 - \lambda(\omega^i)) \beta E_t W_{t+1} \right\}. \tag{16}$$

Note that we focus on choices of P^i such that $\beta E_t J_{t+1} - P - \beta E_t W_{t+1} > 0$.

Each period the total adult population of the economy increases by μQ_t. These new members of the adult population must decide where to search for a home. We assume that once they decide to search in a location, they commit to eventually buy a house and remain there until they receive a taste shock. When they move to a given location, they must search for a house for at least one period, during which time they rent. We assume that each member of the new population initially has a next best alternative which yields present value utility ε. Here ε is distributed across the new members of the population according to a stationary distribution function, $G(\varepsilon)$, with support $[0, \bar{\varepsilon}]$.

There is a critical agent with relative preference, ε_t^c, who is just indifferent between their next-best alternative and becoming a potential buyer

$$\varepsilon_t^c = \beta E_t W_{t+1} \tag{17}$$

The stock of perpetual renters is assumed to grow at a constant rate μ and so is a constant proportion of the total population given by $f = F_t/Q_t$.

4 Equilibrium

A *symmetric equilibrium* is a sequence $\{J_t,\ V_t, W_t,\ P_t,\ \varepsilon_t^c, B_t,\ N_t, H_t,\ \omega_t,\ L_t, l_t,\ w_t\}_{t=0}^{\infty}$ such that,given the evolution of $\{u_t,\ Q_t\}_{t=0}^{\infty}$:

• Given house prices and wages of construction workers, there is free entry into construction:

$$\beta E_t \tilde{V}_{t+1} \leq \frac{w_t}{\phi} + \bar{q}, \qquad H_{t+1} \geq H_t \qquad\qquad \text{w.a.l.o.e.} \tag{18}$$

• The market for construction workers clears:

$$L_t = (N_t + B_t + F_t)\, l_t. \tag{19}$$

• Buyers and sellers are matched according to the matching function (48)
• Given prices, P_t, and market tightness, ω_t the value of being a homeowner satisfies (9).
• Free entry of sellers implies

$$\gamma(\omega_t^i) P_t^i + (1 - \gamma(\omega_t^i))\beta E_t V_{t+1} = V_t \quad \forall i \tag{20}$$

• Free entry of buyers implies

$$u_t^R + \lambda(\omega_t^i)\left(\beta E_t J_{t+1} - P_t^i\right) + (1 - \lambda(\omega_t^i))\beta E_t W_{t+1} = W_t \quad \forall i \tag{21}$$

- New households enter the market optimally so that (17) and (28) are satisfied.
- The stocks of rental versus owned housing must be such that

$$r_t - m + \beta E_t \widetilde{V}_{t+1} = V_t$$

$$H_t^R = B_t + F_t$$

- Boundary conditions on value function rule out bubbles.

In equilibrium, (20) implies a mapping between the market tightness and the price across sub-markets:

$$\gamma\left(\omega(P)\right) = \frac{V_t - \beta E_t V_{t+1}}{P - \beta E_t V_{t+1}}. \tag{22}$$

This condition implies that the probability that a vacant house is sold is lower in submarkets with higher prices. Differentiating yields

$$\omega'(P) = -\frac{\gamma\left(\omega(P)\right)}{\gamma'(\omega)\left(P - \beta E_t V_{t+1}\right)} < 0. \tag{23}$$

Since there is a one-to-one mapping between prices and tightness, from now on we will index sub-markets by P.

Similarly, (21) implies another mapping between the market tightness and the price across sub-markets:

$$\lambda(\omega(P)) = \frac{W_t - u_t^R - \beta E_t W_{t+1}}{\beta E_t J_{t+1} - P - \beta E_t W_{t+1}}. \tag{24}$$

Differentiating yields

$$\omega'(P) = \frac{\lambda(\omega(P))}{\lambda'(\omega)\left(\beta E_t J_{t+1} - P - \beta E_t W_{t+1}\right)} < 0. \tag{25}$$

In equilibrium prices in each submarket are such that it is not possible to create a new submarket, which would attract buyers and in which vacant houses yield strictly higher expected utility. This implies a point of tangency between (22) and (24). At the point of tangency we have

$$\frac{\beta E_t J_{t+1} - P - \beta E_t W_{t+1}}{P - \beta E_t V_{t+1}} = -\left.\frac{\lambda(\omega(P))}{\lambda'(\omega)} \middle/ \frac{\gamma\left(\omega(P)\right)}{\gamma'(\omega)}\right. \tag{26}$$

The total surplus from a housing transaction is $\beta E_t J_{t+1} - \beta E_t W_{t+1} \beta E_t V_{t+1}$. We can therefore express the left hand side of (26) as $s(P)/(1 - s(P))$, where $s(P)$ denotes the buyer's share of the surplus in sub-market P. The right hand side of (26) is equal to the ratio of the elasticity of the matching function w.r.t. the number of buyers to that w.r.t. to the number of sellers. It follows that

Proposition 1 (Moen, 1997): *In a competitive search equilibrium the share of the surplus from house transactions that accrues to the buyer (seller) in each sub-market equals the elasticity of the matching function w.r.t. the number of buyers (sellers) in that sub-market:*

$$s(P) = \epsilon(\omega(P)) = \frac{B}{M}\frac{\partial M}{\partial B}$$

T*he value of* P *which satisfies this is unique. Consequently, only one sub-market opens in equilibrium.*

In equilibrium the stocks of rental and ownable housing is such that the return to renting a house for a period equals the expected gain from holding it vacant and for sale

$$r_t - m = \gamma_t \left(P_t - \beta E_t V_{t+1}\right) \tag{27}$$

Potential buyers at date $t+1$ consist of those at date t who did not find a house in the previous period and new members of the outside population whose alternative is below, ε_t^c. That is

$$B_{t+1} = G(\varepsilon_t^c)\mu Q_t + (1-\lambda_t)B_t. \tag{28}$$

Note that even if ε_t^c were constant over time, the stock of potential buyers, B_t, would grow because Q_t is increasing. Moreover, provided there is sufficient housing available, it follows that the measure of homeowners evolves according to

$$N_{t+1} = (1-\pi)N_t + \lambda_t B_t. \tag{29}$$

We focus on an equilibrium in which population growth is sufficient to ensure that construction of houses is always positive. It follows from (10), (4) and (19) that the quantity of new housing constructed in period t is given by

$$H_{t+1} - H_t = \phi\left(N_t + B_t + F_t\right)\zeta w_t^{\eta} \tag{30}$$

Since $H_{t+1} > H_t$ it follows from (18) that

$$H_{t+1} - H_t = \phi^{1+\eta}\left(N_t + B_t + F_t\right)\zeta\left(\beta E_t V_{t+1} - \bar{q}\right)^{\eta}. \tag{31}$$

Combining (17) and (28) we can express the evolution of the measure of potential buyers as

$$B_{t+1} = G\left(\beta E_t W_{t+1}\right) \mu Q_t + (1 - \lambda_t) B_t. \tag{32}$$

Here we can see that how the entry of potential buyers depends on their sensitivity to the expected value of searching which is determined by the shape of $G(\cdot)$.

To obtain a stationary representation we normalize the state variables by dividing by the total population of the economy, Q_t. Using lower case letters to represent per capita values, it follows that the dynamic equations for potential buyers per capita, owners per capita and houses per capita, respectively, can be written as

$$(1+\mu) b_{t+1} = \mu G\left(\beta E_t W_{t+1}\right) + (1 - \lambda(\omega_t)) b_t \tag{33}$$

$$(1+\mu) n_{t+1} = (1-\pi) n_t + \lambda(\omega_t) b_t \tag{34}$$

$$(1+\mu) h_{t+1} = h_t + \zeta \phi^{1+\eta} \left(n_t + b_t + f\right) \left(\beta E_t V_{t+1} - \bar{q}\right)^{\eta} \tag{35}$$

It also follows that the tightness of the housing market can be expressed as

$$\omega_t = \frac{b_t}{h_t - h_t^R - n_t} \tag{36}$$

and market clearing in the rental market implies

$$h_t^R = b_t + f$$

5 Stationary Equilibrium

In a stationary equilibrium there are no shocks so that $y_t = \bar{y}$. Given stationary values for W and ω, the fraction of the total population who are potential buyers each period is given by

$$b^* = \frac{\mu G\left(\beta W^*\right)}{\mu + \lambda(\omega^*)} \tag{37}$$

From (34) it follows that the steady-state fraction of the total population located in the city is

$$n^* = \frac{\lambda(\omega^*)}{\mu + \pi} b^*. \tag{38}$$

and from (35), the housing stock per capita is

$$h^* = \frac{\zeta \phi^{1+\eta} \left(n^* + b^* + f\right)}{\mu} \left(\beta V^* - \bar{q}\right)^{\eta} \tag{39}$$

Lemma 1: *In a stationary equilibrium, there exists a negative supply-side relationship between the value of a house for sale and market tightness given by*

$$V^* = V^S(\omega^*) = \frac{1}{\beta}\left[\frac{\mu}{\zeta\phi^{1+\eta}}\left(\frac{\mu+\pi+\gamma(\omega^*)}{(\mu+\pi)\,\omega^*+\gamma(\omega^*)}\right)\right]^{\frac{1}{\eta}} + \frac{\bar{q}}{\beta} \tag{40}$$

This relationship can be understood as follows. As the value of vacant housing rises, new construction is stimulated and more houses become available for sale. This drives down the ratio of potential buyers to houses for sale, ω. This result relies on the fact that $\omega\gamma'(\omega)/\gamma < 1$.

In the stationary equilibrium, the values of owners, potential buyers and vacant houses and the steady state, must satisfy the stationary equations

$$J^* = \bar{u}^H + \beta\pi Z + \beta\pi V^* + \beta(1-\pi)J^* \tag{41}$$

$$W^* = \bar{u}^R + \lambda(\omega^*)\beta(J^* - P^*) + (1-\lambda(\omega^*))\beta W^* \tag{42}$$

$$V^* = \gamma(\omega^*)\,P^* + (1-\gamma(\omega^*))\beta V^* \tag{43}$$

$$P^* = \beta(1-s(\omega^*))\,(J^* - W^*) + \beta s(\omega^*)V^* \tag{44}$$

$$r^* = m + \gamma(\omega^*)\,[P^* - \beta V^*] \tag{45}$$

One can solve the first four equations of this system for J^*, W^*, V^* and P^*. After some manipulation, this yields another relationship between the value of houses for sale and market tightness:

Lemma 2: *In a stationary equilibrium there exists a positive demand–side, relationship between V^* and ω^* given by*

$$V^* = V^D(\omega^*) = \frac{\beta(1-s(\omega^*))\gamma(\omega^*)\left(\bar{u}^H - \bar{u}^R\right)}{(1-\beta)\,\beta(1-s(\omega^*))\gamma(\omega^*) + (1-\beta+\beta\pi)\,(1-\beta+\beta s(\omega^*)\,\lambda(\omega^*))} \tag{46}$$

Intuitively, a higher ratio of buyers to sellers has two effects. Firstly it increases the rate at which houses will sell, γ, which for a given selling price drives up the value of a vacant house. It also lowers the rate at which houses are found which increases the gain from becoming an owner. This raises the selling price of houses which also drives up the value of a house for sale. Note that

$$\bar{u}^H - \bar{u}^R = z^H - z^R + r^*$$

where r^*

Proposition 1: *Under certain restrictions on parameters, there exists a unique steady state equilibrium.*

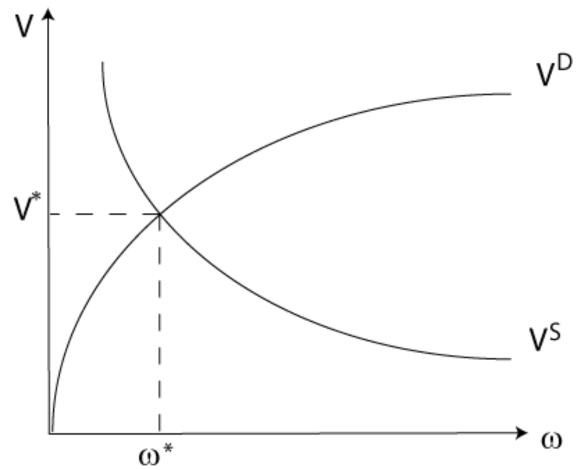


Figure 3: Steady State Equilibrium

Figure 3 depicts the stationary equilibrium at the intersection of (40) and (46).

6 Extensions

6.1 Endogenous Land Price

In our basic model, unit land prices are held fixed. It is straighforward to extend the model so that land prices are endogenous. Specifically, we suppose the unit land price is an increasing function of the stock of housing:

$$q_t = q(h_t) = \bar{q} h_t^{\xi}. \tag{47}$$

7 Calibration

7.1 General Parameters

In this section we discuss the calibration scheme. We define a period to equal one quarter. We set β to reflect an annual interest rate of 4% and μ is chosen to match annual population growth during the 1990s.[7] We normalize so that $\bar{y} = 1$. Thus present values and prices are all measured relative to the stationary income. We set η to match typical estimates of labour supply elasticity (e.g. Altonji, 1986). The parameter ϕ represents the labor productivity of

[7] Population growth has slowed somewhat in recent years.

the construction sector. The rato of permits issued in the US each quarter to the numbers of employees in residential construction is approximately 0.1. If the average working week is 35 hours this amounts to about 400 hours per quarter, which yields the number of permits produced per hour worked equal to about 0.00025. The unit price of land, $\bar{q}$, is set so that the relative share of land in the price of housing is 40% (see Davis and Palumbo, 2008). The average price of a house is approximately is 3 times annual income or 12 times quarterly income.

Table 3: General Parameters

Parameter	Value	Target
β	0.99	Annual real interest rate = 4%
μ	0.003	Annual population growth rate = 1.2%
η	0.25	Elasticity of labour supply = 0.25
ϕ	0.00025	Quarterly permits/construction employment (hours)
$\bar{q}$	4.8	Average land price-income ratio
α	4	Elasticity of cross-city income distribution at mean
m	0.0233	Average rent – average income ratio, $r^* = 0.14$
$z^H - z^R$	0.03	Monthly housing premium = 1%

The only variable in steady state that depends on $G(\cdot)$ is the measure of searching households per capita, b^*. This is not something that is likely to be directly observable and so these parameters are hard to identify. The dynamics of the model depend crucially on the shape of $G(\cdot)$ in the vicinity of ε^c since this determines the responsiveness of new entrants to changes in the value of search. Specifically, in our dynamic analysis this responsivenes depends on the elasticity of the distribution evaluated at ε^c:

$$\alpha = \frac{\varepsilon^c G'(\varepsilon^c)}{G(\varepsilon^c)}.$$

One approach to calibration is to use the elasticity of the distribution of average incomes across cities. The assumption is that the value of living in a city is proportional to income. In the steady state of our model this would be true if the only exogenous factor that varied across cities were average incomes. Figure 4 depicts the cumulative distribution of per capita incomes across all MSAs in the US. We compute the arc elasticity between two points on this distribution that are half a standard deviation apart and equidistant from the mean. This resulted in a value of $\alpha \simeq 4$. Of course, there is considerable uncertainty as to whether this is a reasonable value, so we consider the sensitivity of our results to changes in it.

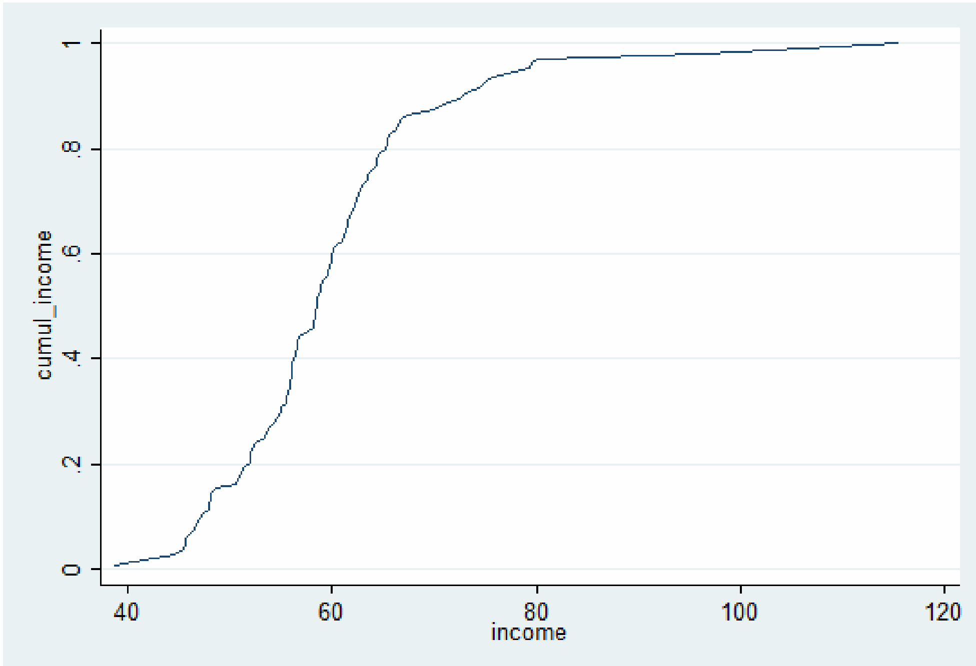


Figure 4: Distribution of per capita incomes across cities in 2000 ($000s)

7.2 Cobb–Douglas matching

Suppose the matching function takes a simple Cobb-Douglas form given by

$$M_t = \kappa B_t^{\delta} S_t^{1-\delta}, \tag{48}$$

where $\kappa > 0$ and $\delta \in (0, 1)$. In this case, $\epsilon(\omega) = \delta$ so the share of the surplus accruing to each party in a housing transaction is constant. This implies that the competitive search model is equivalent to a random search model with Nash Bargaining in which the Hosios condition is assumed. In particular, the share of the surplus accruing to each party is not sensitive to the tightness of the market.

We choose the remaining parameters so that several key steady state statistics match their average counterparts in US data. In particular, κ, δ, π and ζ jointly determine the steady state values of the house price, the vacancy rate, the average time it takes to buy a house and the average time it takes to sell a house. We set the price of a house to be 3 times annual income or 12 times quarterly income. We also assume that, in steady state, the time taken to sell a house is equal to the time taken to buy. This is consistent with the findings of Diaz and Jerez (2010).

Average vacancy rates for the US economy and by MSA are available from the Census Bureau's Housing Vacancy Survey (HVS). In our model, houses that are vacant in equilibrium are designated to be for sale. The HVS distinguishes the category "vacant units which are for sale only". In 2000, for example, this category constituted 1% of the overall housing stock. Since owned homes constituted approximated two-thirds of the housing stock, this corresponds to a home-owner vacancy rate of about 1.5%.[8]

However, housing units that are in the category "vacant units for rent" actually consist of vacant units offered for rent only and those offered *both* for rent and sale. In 2000, for example. this category constituted a further 2.5% of the overall housing stock.[9] In our model, vacant units are technically available for rent in the subsequent period, so it would make sense to include those vacant units offered for both rent and sale. In addition, only about half of all vacant units are included in either of the categories (i.e. for sale only or for rent or sale). The remainder include units that are held off the market for various other reasons. For example, this category includes vacant units located in a multi-unit structure which is for sale. For these reasons, we consider "high vacancy rate" case, where we assume an additional 1% of the housing stock is vacant and for sale. This corresponds to a home-owner vacancy rate of about 3%.

Under the assumption that $\omega = 1$, in the steady state, $h - h^R = n + b$. It follows that the home-owner vacancy rate is

$$v = \frac{h - h^R - n}{h - h^R} = \frac{b}{n + b}. \tag{49}$$

Using (38), this implies that

$$\gamma^* = \lambda^* = (\mu + \pi)\,\frac{1 - v}{v}. \tag{50}$$

Given the value of μ from Table 1 and each of our targets for v, we choose π so that $\gamma = \lambda = 0.75$. This implies that the average time on the market is about 4 months.

This may seem somewhat high given that according to the National Association of Realtors, the time taken to sell a typical house is about 8 weeks.[10] This estimate of "time on the market", however, is potentially misleading because houses may sometimes be strategically

[8] This number is close to the average over the period 1980-2008. However, more recently homeowner vacancy rates have exceeded 2.5%

[9] Again, since rental units constitute about a third of the housing stock, this corresponds to a rental vacancy rate of about 8%.

[10] There are varying estimates of the time to buy and the time to sell. Diaz and Jerez (2008) use 2 months based on a report from the National Association of Realtors. Piazzesi and Schneider (2009) suggest using 6 months. Anglin and Arnott (1999) report estimates of up to 4 months.

de-listed and quickly re-listed in order to reset the "days on market" field in the MLS listing. In their detailed analysis of the housing market in 34 Cook county (Illinois) suburbs over the period 1992-2002, Levitt and Syverson (2008) compute time-to-sale by "summing across all of a house's listing periods that are separated by fewer than 180 days." They estimate that the average time on the market for a house that eventually sells is 94 days (3.07 months). Moreover, in their sample of 127,000 houses, 22% of houses put up for sale never sell. In less active markets it is likely that the time on the market is even longer.

Tables 4.1 and 4.2, give the parameter values implied for each vacancy rate target. Note that the value of δ required to hit these targets, given the other parameters, implies that most of the surplus from housing transactions goes to the seller.

Table 4.1: Parameters — Cobb-Douglas Matching Function, Low Vacancy Rate

Parameter	Value	Target
κ	0.7500	Vacancy rate = 1.5%
δ	0.1864	Months to sell = 4
π	0.0085	Months to buy = 4
ζ	153.22	$P^* = 12$

Table 4.2: Parameters — Cobb-Douglas Matching Function, High Vacancy Rate

Parameter	Value	Target
κ	0.7500	Vacancy rate = 3.0%
δ	0.1188	Months to sell = 4
π	0.0202	Months to buy = 4
ζ	154.85	$P^* = 12$

7.3 Generalized urn-ball matching

For matching functions other than Cobb–Douglas the equilibrium share of the surplus received by the buyers and sellers are not generally constant. Here we consider an alternative matching function given by

$$M(B,S) = S\varphi(1 - e^{-\theta \frac{B}{S}})$$

If $\theta = 1$, the matching probabilities are equivalent to the "urn-ball" matching process assumed by Diaz and Jerez (2009). Here we assume a more general matching function in order to calibrate the model to the same targets as before. This generalization could be motivated along the lines of Albrecht, Gauthier, and Vroman (2003), where θ denotes the average number of applications to purchase made per period and φ indexes the effort required to process each application. Tables 5.1 and 5.2 contain the parameter values needed to achieve the same targets as before for each vacancy rate case.

Table 5.1: Parameters — Urn-Ball Matching Function, Low Vacancy Rate

Parameter	Value	Target
φ	0.8061	Vacancy rate = 1.5%
θ	2.7610	Months to sell = 4
π	0.0085	Months to buy = 4
ζ	153.22	$P^* = 12$

Table 5.1: Parameters — Urn-Ball Matching Function, High Vacancy Rate

Parameter	Value	Target
φ	0.7765	Vacancy rate = 3.0%
θ	3.3840	Months to sell = 4
π	0.0202	Months to buy = 4
ζ	154.85	$P^* = 12$

The surplus accruing to the buyer for the urn-ball matching function is

$$s = \epsilon(\omega) = \frac{\theta\omega}{e^{\theta\omega} - 1},$$

which is decreasing in market tightness, ω. That is, as the ratio of buyers to sellers increases, the share received by buyers falls.

7.4 No housing market frictions

As a benchmark, it is useful to compare our results to those from an economy with no frictions in the housing market. In this economy there is no distinction between renting and owning — new entrants can either rent or purchase a house immediately and move in. Since households derive more utility from owning and construction costs are the same, the only rental that will occur in equilibrium will be that by the perpetual renters. With no frictions, the dynamic system can be written as

$$
\begin{aligned}
(1+\mu)b_{t+1} &= \mu G\left(\beta E_t J_{t+1} - P_t\right) && (51)\\
(1+\mu)n_{t+1} &= (1-\pi)n_t + b_t && (52)\\
(1+\mu)h_{t+1} &= h_t + \zeta\phi^{1+\eta}\left(n_t + b_t + f\right)\left(\beta E_t P_{t+1} - \bar{q}\right)^{\eta} && (53)\\
h_t &= n_t + b_t + f && (54)\\
J_t &= u_t^H + \beta\pi\left(Z + P_t\right) + \beta(1-\pi)E_t J_{t+1} && (55)
\end{aligned}
$$

The economy with no frictions is comparable to the model discussed by Glaeser and Gyourko (2008). An important difference is that they assume the outside alternative to living in a the city yields a homogeneous payoff. This effectively implies immediate entry of buyers until the price of housing adjusts enough to keep the value of entering constant. This tends to generate high variance in both prices and construction in response to income shocks. In our model there is a distribution of alternatives, so that the critical ouside value rises helping to stem the flow into the city. We can replicate Glaeser and Gyourko's equilibrium by assuming a value of α which is very high.

In the stationary equilibrium with no search frictions, the price is simply

$$P^* = \frac{1}{\beta}\left[\frac{\mu}{\zeta\phi^{1+\eta}}\right]^{\frac{1}{\eta}} + \frac{\bar{q}}{\beta} \tag{56}$$

Given the parameters in Table 6, we use this to derive the value of ζ such that $P^* = 12$.

8 Qualitative Dynamics

We log–linearize the system around the steady–state which reduces it to a system of first–order linear difference equations. One can show that this system satisifes the conditions for saddle-path stability. We numerically solve for the implied dynamics using a Generalized Schur decomposition due to Klein (2007).

In this section, we use our calibrated economy to illustrate the key qualitative features of the model's dynamics. To do so we assume that the process followed by the log of income, $\ln y_t$, is a simple AR(1) process with persistence parameter $\rho = 0.98$ and standard deviation $\sigma_\varepsilon = 0.01$. We use this example to illustrate that the model's dynamics are not driven by the dynamics of income (although, as we will see below, the hump-shaped dynamics of income magnify the effects to some extent). Note that in an asset pricing framework with no frictions such persistence cannot translate into momentum in house price changes. Indeed the impulse response of house prices simply inherits the shape of that for housing dividends.[11] In our model with frictions this is not generally the case.

The implied impulse response functions following a shock to income illustrated in Figures 5 and 6 for the low and high steady–state vacancy rate scenarios. Each figure depicts the IRFs for prices, population, vacancies and market tightness for the model with no search frictions and with search frictions under the two alternative matching functions.

In both scenarios and for both matching functions, the housing market frictions act so as to generate a hump–shaped IRF for house prices. There are three key forces at play here:

(1) The initial rise in the value of living in a city generates an immediate increase in search activity. However, it takes time for potential buyers to match with a house. Although sales and the probability of selling rise immediately, construction of new housing takes time to respond. Even if the value of searching subsequently declines (due to mean reversion), the number of searchers continues to rise and the stock of vacancies declines. Consequently, the ratio of buyers to sellers continues to rise in the short term, further driving up the rate at which houses sell and hence the value of a vacant house. Since, in equilibrium, house prices partly reflect this value, they rise too. Eventually, the stock of potential buyers starts to fall as they are absorbed into the owner-occupied housing market, and the decline in vacancies slows (and eventually reverses) as construction rates catch up. This causes the ratio of buyers to sellers to decline and prices eventually start to revert back towards their steady state values.

(2) Given that prices are expected to rise initially in response to the shock, there is an increase in the measure of unoccupied houses which are rented rather than put up for sale immediately. This increased relative supply of rental housing keeps the rental rate from rising too rapidly and induces the entry which drives the subsequent price appreciation. This effect tends to magnify the underlying momentum in house prices. The larger the initial (steady state) vacancy rate, the bigger this effect is. This can be seen by comparing the house price movements with low and high vacancy rates. In the high vacancy rate case, the price momentum and overall variance is much greater.

(3) In the urn-ball matching function case, the share of the surplus received by the buyer falls as the buyer seller ratio rises.

[11] Glaeser and Gyourko (2006) consider an ARMA(1,1) process for u_t, but house price still mean revert very quickly.

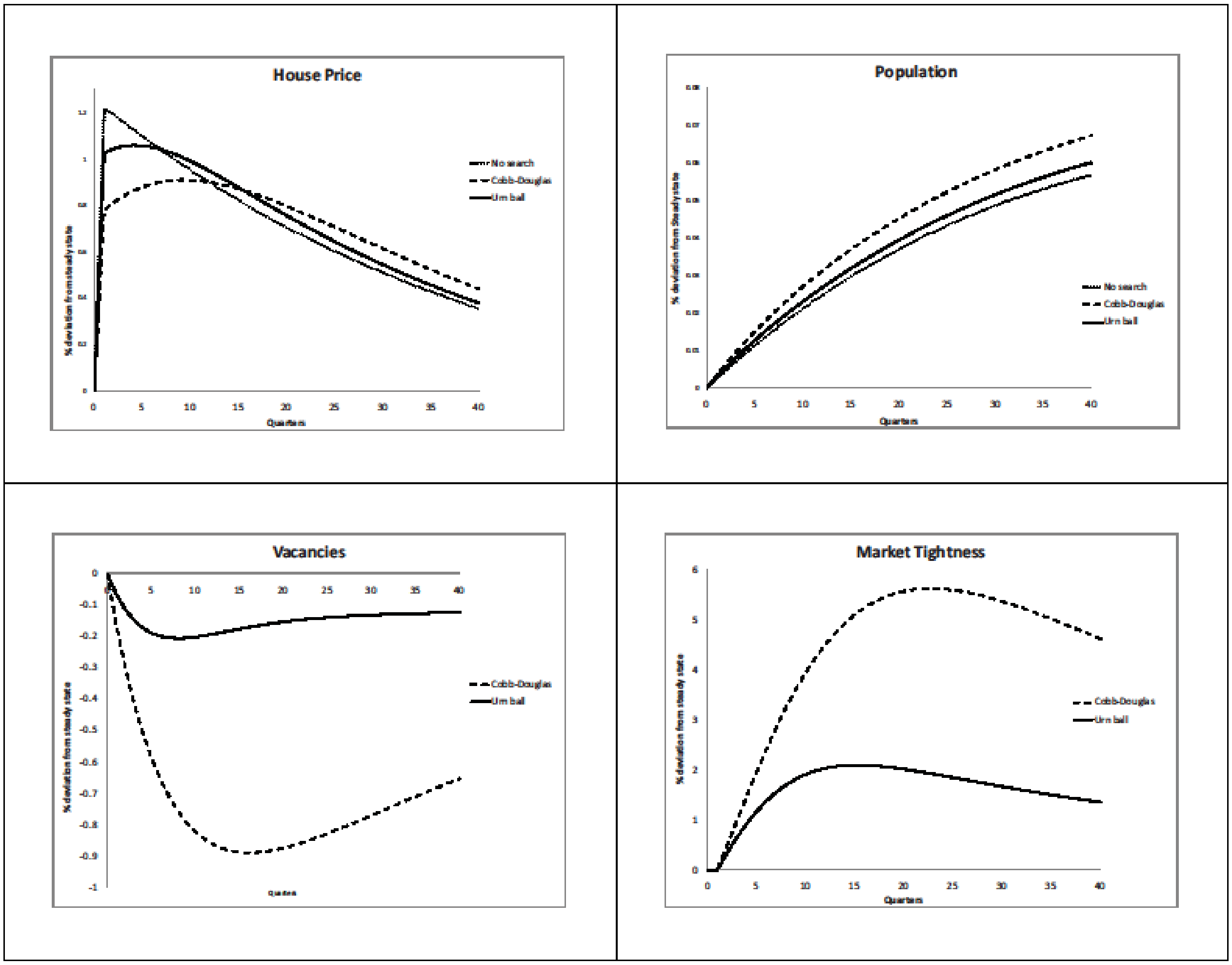


Figure 5: IRFs: AR(1) process with low vacancy rate

9 Quantitative Evaluation

So far we have used our calibrated model largely for illustrative purposes, given an arbitrary shock process. But how does the model perform for a quantitatively relevant shock process? To address this question requires us to compare the model output with the stylized facts reported in Section 2. However, there is an important issue to address first: there is a mismatch between the frequency of available city level income data and the period length assumed in our calibrated model. The former is only available annually, whereas the latter is calibrated to a quarterly frequency. One option would be to increase the length of a model period to a year. However, this would imply that houses would remain vacant for at least a

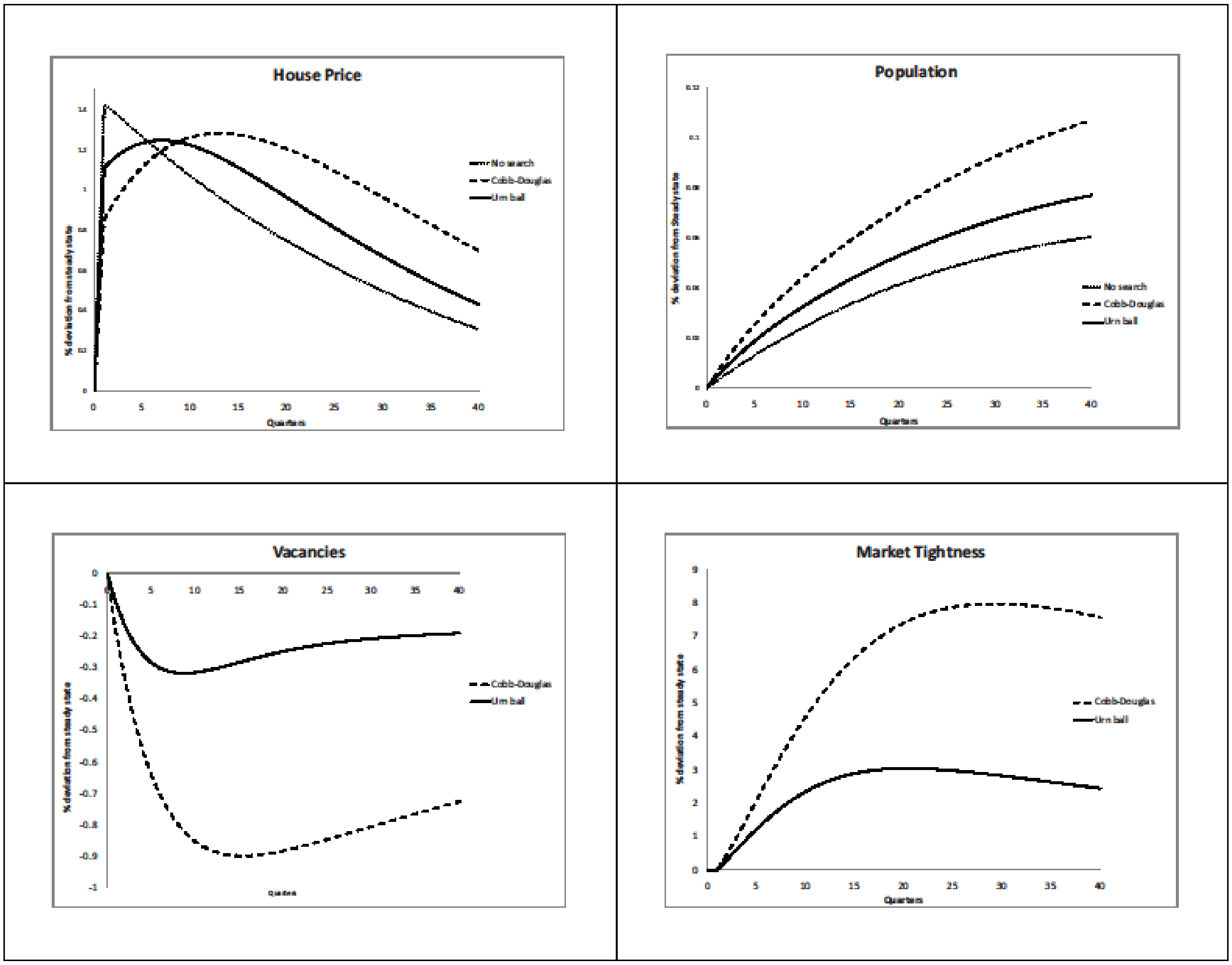


Figure 6: IRFs: AR(1) process with high vacancy rate

year, which is clearly counterfactual.

The approach taken here is to derive a quarterly process for income that has the same key properties at annual frequencies as the data (see appendix). Using this together with the transition function derived from the linearized model, we generated sample paths for the key variables of the model. We used these to generate annual sample paths from which we computed the key annual statistics documented in Tables 6.1 and 6.2. These are compared to the same statistics for the US economy based on the SVECM discussed in Section 2.

Figure 7 shows the implied IRFs for the high vacancy rate case. Quantitatively, the IRFs are much the same as in the AR(1) case. However, because of the hump-shaped process followed by incomes in the data, both the momentum and mean-reversion in the model with search frictions is greater.

Table 6.1: Moments from the model with realistic income shocks ($v = 1.5\%$)

	σ_p/σ_y	σ_n/σ_y	σ_{py}	σ_{ny}	ρ_1	ρ_2	ρ_3	ρ_4
US Data (1980-2008)	1.6086	0.4343	0.8775	0.8453	0.7921	0.3872	0.0243	-0.2201
US Data (1980-2000)	1.5266	0.7038	0.9231	0.6216	0.7379	0.4224	0.1874	0.0370
$\alpha = 1$								
No search	0.5454	0.0567	0.7753	0.6767	0.0247	-0.0520	-0.0510	-0.0442
Cobb-Douglas	0.4366	0.0764	0.8386	0.5649	0.4832	0.2723	0.1323	0.0357
Urn ball	0.4409	0.0675	0.8491	0.5802	0.2513	0.0693	-0.0136	-0.0548
$\alpha = 2$								
No search	0.7767	0.0939	0.8066	0.6335	0.0249	-0.0519	-0.0487	-0.0468
Cobb-Douglas	0.6834	0.1239	0.8679	0.5474	0.2519	0.1149	0.0407	-0.0068
Urn ball	0.6945	0.1098	0.8747	0.5345	0.1451	0.0185	-0.0324	-0.0474
$\alpha = 4$								
No search	1.0213	0.1442	0.8497	0.5628	0.0238	-0.0468	-0.0429	-0.0403
Cobb-Douglas	0.9529	0.1906	0.8930	0.5066	0.1654	0.0531	-0.0009	-0.0275
Urn ball	0.9609	0.1758	0.8946	0.4902	0.0976	-0.0079	-0.0399	-0.0483
$\alpha = 8$								
No search	1.2337	0.2108	0.8917	0.4824	0.0192	-0.0532	-0.0399	-0.0382
Cobb-Douglas	1.1973	0.2845	0.9176	0.4497	0.1168	0.0173	-0.0206	-0.0433
Urn ball	1.2181	0.2547	0.9266	0.4103	0.0810	-0.0283	-0.0488	-0.0464

Table 6.2: **Moments from the model with realistic income shocks** ($v = 3\%$)

	σ_p/σ_y	σ_n/σ_y	σ_{py}	σ_{ny}	ρ_1	ρ_2	ρ_3	ρ_4
US Data (1980-2008)	1.6086	0.4343	0.8775	0.8453	0.7921	0.3872	0.0243	-0.2201
US Data (1980-2000)	1.5266	0.7038	0.9231	0.6216	0.7379	0.4224	0.1874	0.0370
$\alpha = 1$								
No search	0.5454	0.0567	0.7753	0.6767	0.0247	-0.0520	-0.0510	-0.0442
Cobb-Douglas	0.8257	0.1470	0.7906	0.5778	0.7033	0.4291	0.2393	0.1035
Urn ball	0.6618	0.0895	0.7980	0.6497	0.4224	0.1857	0.0515	-0.0299
$\alpha = 2$								
No search	0.7767	0.0939	0.8066	0.6335	0.0249	-0.0519	-0.0487	-0.0468
Cobb-Douglas	1.0362	0.1867	0.8347	0.5805	0.3717	0.1970	0.0870	0.0209
Urn ball	0.8859	0.1255	0.8277	0.6222	0.2193	0.0588	-0.0101	-0.0444
$\alpha = 4$								
No search	1.0213	0.1442	0.8497	0.5628	0.0238	-0.0468	-0.0429	-0.0403
Cobb-Douglas	1.2343	0.2327	0.8696	0.5568	0.2167	0.0865	0.0247	-0.0133
Urn ball	1.2259	0.2328	0.8694	0.5567	0.2220	0.0900	0.0286	-0.0122
$\alpha = 8$								
No search	1.2337	0.2108	0.8917	0.4824	0.0192	-0.0532	-0.0399	-0.0382
Cobb-Douglas	1.3888	0.2890	0.9020	0.5060	0.1547	0.0384	-0.0066	-0.0303
Urn ball	1.3830	0.2928	0.8994	0.5108	0.1467	0.0403	-0.0051	-0.0372

10 Concluding Remarks

Qualitatively, adding competitive search into a dynamic model of housing markets with endogenous entry and construction helps us rationalize movements in house prices. However, for a matching function that implies constant elasticities with respect to tightness, adding search frictions tends to dampen the volatility of prices. Allowing for alternative matching functions such as the urn ball variety, which implies the surplus share depends on market tightness, retains the basic shape of the response of prices but implies greater volatility. A calibrated version of our model captures the qualitative movements in the date quite well, but generally understates them quantitatively.

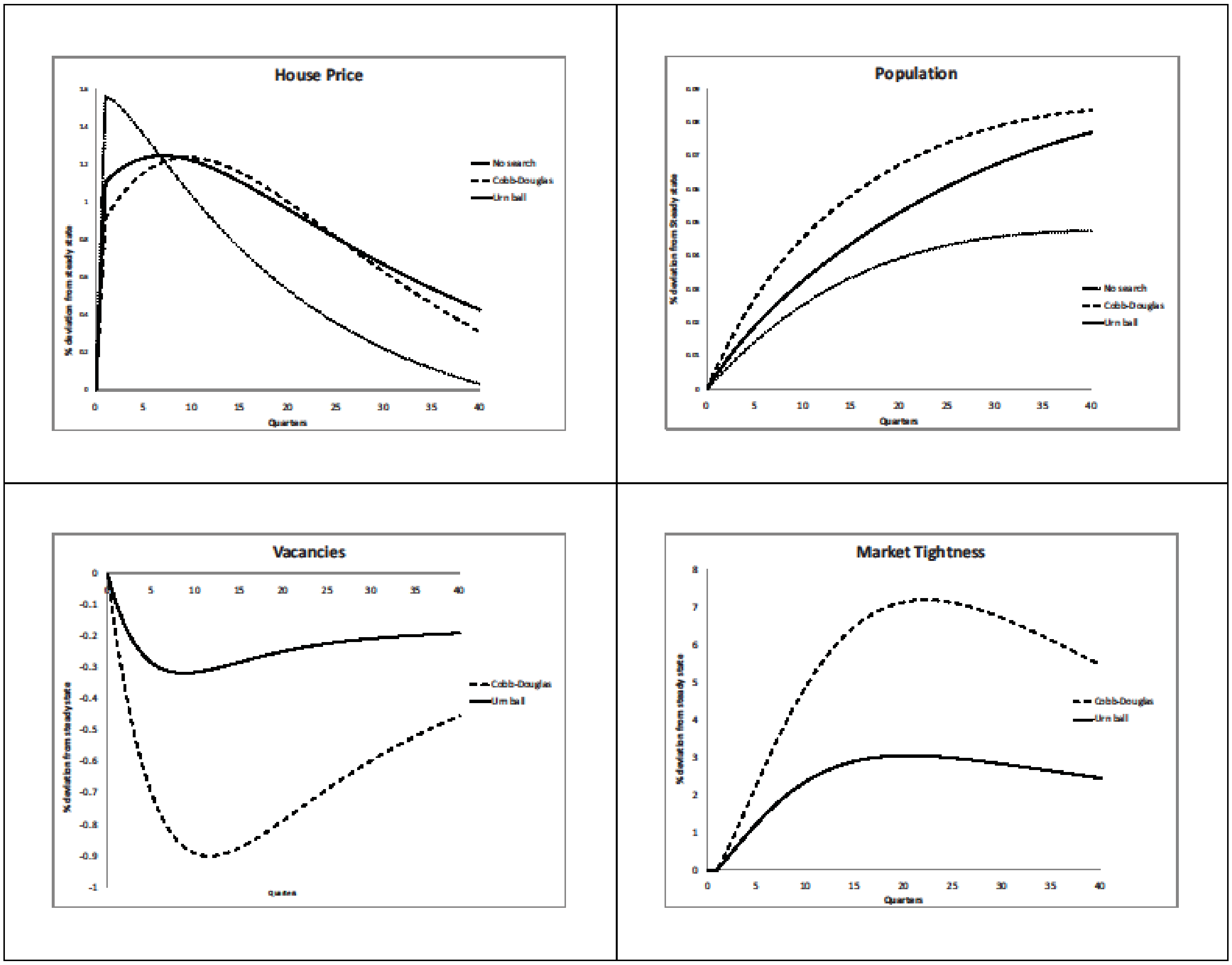


Figure 7: IRFs: Realistic income shocks process, high vacancy rate, $\alpha = 4$

11 Appendix

11.0.1 Proofs and Derivations

Proof of Proposition 1: Equation (26) can be expressed as

$$\begin{aligned}\frac{s}{1-s} &= \frac{M(B,S)}{B\frac{\partial M}{\partial S}\left(\frac{S}{B}\right)^2} \Bigg/ \frac{M(B,S)}{S\frac{\partial M}{\partial B}} \\ &= \frac{\frac{B}{M}\frac{\partial M}{\partial B}}{\frac{S}{M}\frac{\partial M}{\partial S}} = \frac{\epsilon(\omega)}{1-\epsilon(\omega)}\end{aligned}$$

The result follows. Note that $s'(P) < 0$ and $\omega'(P) < 0$. If $\epsilon'(\omega) < 0$, the point of tangency must be unique.

Proof of Lemma 1: Substituting (37), (38), (39) into (36) yields

$$\omega^* = \frac{b^*}{h^* - n^*} \tag{57}$$

$$\omega^* = \frac{\mu + \pi}{\frac{\zeta\phi^{1+\eta}}{\mu}(\mu + \pi + \lambda)(\beta V^* - \bar{q})^\eta - \lambda(\omega^*)} \tag{58}$$

$$\frac{\zeta\phi^{1+\eta}}{\mu}(\beta V^* - \bar{q})^\eta = \frac{\mu + \pi + \gamma(\omega^*)}{(\mu + \pi)\,\omega^* + \gamma(\omega^*)} \tag{59}$$

Re-arranging yields (43). The the sign of the derivative of $V^S(\cdot)$ is a with respect to ω depends on the sign of

$$\begin{aligned} & ((\mu+\pi)\,\omega^* + \gamma(\omega^*))\,\gamma'(\omega^*) - (\mu+\pi+\gamma(\omega^*))\,((\mu+\pi)+\gamma'(\omega^*)) \\ = \ & (\mu+\pi)\,[\omega^*\gamma'(\omega^*) - \gamma(\omega^*) - \mu - \pi - \gamma'(\omega^*)] \end{aligned}$$

A sufficient condition for this to be negative is $\omega^*\gamma'(\omega^*) < \gamma(\omega^*)$. This must be true for any CRS matching function, since it implies that $\gamma(\omega)$.is homogenous of degress less than 1.

Proof of Lemma 2: We can express (42) - (43) as

$$[1 - \beta(1-\pi)]\,J^* = \bar{u}^H + \beta\pi Z + \beta\pi V^* \tag{60}$$

$$[1 - (1 - \lambda(\omega^*))\beta]\,W^* = \bar{u}^R + \lambda(\omega^*)\beta J^* - \lambda(\omega^*)\beta P^* \tag{61}$$

$$P^* = \beta(1-s)J^* - \beta(1-s)W^* + \beta s V^* \tag{62}$$

$$P^* = \frac{[1 - (1 - \gamma(\omega^*))\beta]}{\gamma(\omega^*)} V^* \tag{63}$$

Substituting out P^* yields

$$J^* = \frac{\bar{u}^H + \beta\pi Z + \beta\pi V^*}{1 - \beta(1-\pi)} \tag{64}$$

$$[1 - (1-\lambda)\beta]\,W^* = \bar{u}^R + \lambda\beta J^* - \lambda\beta\frac{[1 - (1-\gamma)\beta]}{\gamma} V^* \tag{65}$$

$$[1 - (1-\gamma)\beta - \gamma\beta s]\,V^* = \gamma\beta(1-s)J^* - \gamma\beta(1-s)W^* \tag{66}$$

Substituting out J^* yields

$$W^* = \frac{\bar{u}^R}{[1 - (1-\lambda)\beta]} + \lambda\beta\left(\frac{\bar{u}^H + \beta\pi Z + \beta\pi V^*}{[1 - (1-\lambda)\beta]\,(1 - \beta(1-\pi))}\right) - \lambda\beta\frac{[1 - (1-\gamma)\beta]}{[1 - (1-\lambda)\beta]\,\gamma} \tag{67}$$

$$[1 - (1-\gamma)\beta - \gamma\beta s]\,V^* = \gamma\beta(1-s)\left(\frac{\bar{u}^H + \beta\pi Z + \beta\pi V^*}{1 - \beta(1-\pi)}\right) - \gamma\beta(1-s)W^* \tag{68}$$

Finally, substituting out W^* we have

$$\left[1-(1-\gamma)\beta-\gamma\beta s-\frac{\gamma\beta(1-s)\beta\pi}{1-\beta(1-\pi)}+\frac{\gamma\beta(1-s)\lambda\beta\beta\pi}{[1-(1-\lambda)\beta]\,(1-\beta(1-\pi))}-\gamma\beta(1-s)\lambda\beta\frac{[1-(1-\gamma)\beta]}{[1-(1-\lambda)\beta]\,\gamma}\right]$$

$$= \frac{\gamma\beta(1-s)}{[1-(1-\lambda)\beta]}\left[\frac{\bar{u}^H+\beta\pi\left(\frac{\bar{u}^R}{1-\beta}\right)}{1-\beta+\beta\pi}(1-\beta)-\bar{u}^R\right]$$

Re-arranging yields (46). Dividing the top and bottom by $(1-s(\omega^*))\gamma(w^*)$ etc. yields

$$V^*=\frac{\beta\left(\bar{u}^H-\bar{u}^R\right)}{\frac{(1-\beta)(1-\beta+\beta\pi)}{(1-s(\omega^*))\gamma(\omega^*)}+(1-\beta)\,\beta+(1-\beta+\beta\pi)\,\beta\frac{s(\omega^*)}{(1-s(\omega^*))\omega^*}} \tag{69}$$

Since $\gamma'(w^*)>0$ and $s'(\omega)\leq 0$, this is clearly increasing in ω^*.

Proof of Proposition 2: First observe that since $V^S(\omega)$ is decreasing in ω^* and $V^D(\omega^*)$ is increasing in ω^*, if a steady-state equilibrium exists it must be unique. Existence basically requires that the curves intersect at a value of ω^* such that $\gamma(\omega^*)<1$ and $\lambda(\omega^*)<1$. The implied minimum and maximum values of ω are given by $\lambda(\underline{\omega})=1$ and $\gamma(\overline{\omega})=1$. Then an (interior) equilibrium will exist if $V^D(\underline{\omega})<V^S(\underline{\omega})$ and $V^D(\overline{\omega})>V^S(\overline{\omega})$.

Solving the dynamic system: The dynamic system is given by

$$\begin{aligned}
\ln y_t &= (1-\rho)\ln\bar{y}+\sum_{i=1}^{T}\rho_i\ln y_{t-i}+s_t\\
(1+\mu)\,n_{t+1} &= n_t+\lambda(\omega_t)b_t\\
(1+\mu)\,h_{t+1} &= h_t+\zeta\phi^{1+\eta}\,(n_t+b_t+f)\,(\beta E_tV_{t+1}-\bar{q})^{\eta}\\
(1+\mu)\,b_{t+1} &= \mu G\,(\beta E_tW_{t+1})+(1-\lambda(\omega_t))\,b_t\\
\omega_t &= \frac{b_t}{h_t-b_t-f-n_t}\\
J_t &= y_t+x_t+z^H+\beta\left[\pi\,(Z+E_tV_{t+1})+(1-\pi)E_tJ_{t+1}\right]\\
W_t &= y_t+x_t+z^R-r_t+\lambda(\omega_t)\,(\beta E_tJ_{t+1}-P_t)+(1-\lambda(\omega_t))\beta E_tW_{t+1}\\
V_t &= \gamma(\omega_t)P_t+(1-\gamma(\omega_t))\beta E_tV_{t+1}\\
P_t &= \beta(1-s(\omega_t))E_t\,(J_{t+1}-W_{t+1})+\beta s(\omega_t)E_tV_{t+1}\\
r_t &= m+\gamma(\omega_t)\,(P_t-\beta E_tV_{t+1})
\end{aligned}$$

11.1 Stationary Equilibrium with no search

In a stationary equilibrium there are no shocks so that $u_t^H = \bar{u}^H$. Housing market clearing implies

$$h^* = n^* + b^*$$

and it follows directly that the stationary equilibrium price is

$$P^* = \frac{1}{\beta}\left[\frac{\mu}{\zeta\phi^{1+\eta}}\right]^{\frac{1}{\eta}} + \frac{\bar{q}}{\beta} \tag{70}$$

The value of being a homeowner is then

$$J^* = \frac{\bar{u}^H + \beta\pi Z + \beta\pi P^*}{1-\beta(1-\pi)} \tag{71}$$

Given stationary values for J and P, the new entrants per period is

$$b^* = \frac{\mu}{1+\mu} G\left(\beta J^* - P^*\right) \tag{72}$$

and the steady-state fraction of the total population located in the city is

$$n^* = \frac{1}{\mu+\pi} b^*. \tag{73}$$

Finally, the housing stock per capita is

$$h^* = \frac{\zeta\phi^{1+\eta}\left(n^* + b^*\right)}{\mu}\left(\beta P^* - \bar{q}\right)^{\eta} \tag{74}$$

11.2 House Prices at Quarterly Frequency

Here we use a panel of 45 cities with quarterly data between 1977 and 2008. The first column of Table A1 shows the estimates for the full sample. We found that 5 lags of the growth in prices were necessary and sufficient to describe the evolution of prices.

Table A1: Quarterly Panel (fixed effect) estimates for price process

Parameter	Full Sample (1977-2008) 45 cities	Coastal Cities (1977-2008) 20 cities	Inland Cities (1977-2008) 25 cities	Truncated Sample (1977-2000) 45 cities
α_1	0.09 (6.78)	0.13 (6.43)	-0.01 (0.65)	0.01 (0.64)
α_2	0.21 (16.11)	0.21 (10.52)	0.17 (9.79)	0.17 (11.46)
α_3	0.21 (15.74)	0.13 (6.45)	0.26 (15.37)	0.19 (12.54)
α_4	0.20 (16.11)	0.20 (10.42)	0.24 (14.00)	0.21 (14.86)
α_5	0.05 (4.05)	0.02 (0.89)	0.13 (7.28)	0.07 (5.09)
β	-0.02 (15.74)	-0.03 (10.69)	-0.03 (12.08)	-0.03 (12.66)
City dummies	Yes	Yes	Yes	Yes
Time dummies	Yes	Yes	Yes	Yes
σ_ε	0.016	0.016	0.014	0.017
σ_μ	0.002	0.002	0.002	0.002
ρ	0.012	0.012	0.014	0.011
R^2 within	0.50	0.61	0.42	0.35
R^2 between	0.68	0.34	0.63	0.71
R^2 overall	0.50	0.61	0.43	0.35
# of obs.	5760	2560	3200	4320

Note: t-statistics in parenthesis

Given these estimates we extract an AR(6) process describing the relative price dynamics of the typical city. The implied impulse response function is illustrated in the top panel of Figure 8. As can be seen, the house prices dynamics can be characterized as exhibiting a hump-shaped pattern, with initial autocorrelation and subsequent mean-reversion. The peak in the IRF occurs after about 3 years and after about 6 years the autocorrelation becomes negative.

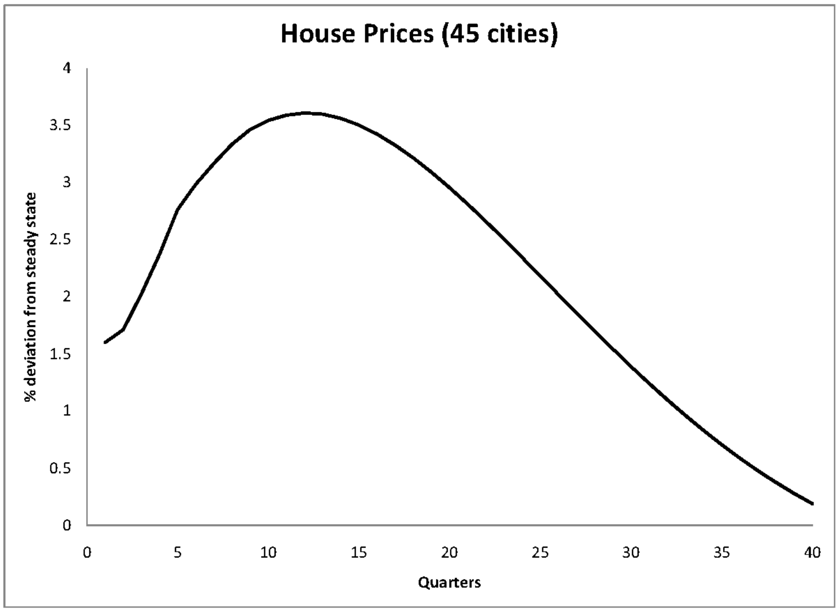
11.3 Translation of the shock process from Annual to Quarterly

If we now think of a period as a quarter, we can write an annual AR(2) process as

$$x_t = b_1 x_{t-4} + b_2 x_{t-8} + \varepsilon_t.$$

Let $y_t = x_{t-4}$. Then we can write this as a stacked system given by

$$\begin{aligned} X_t &= \mathbf{B} X_{t-4} + \mathbf{e}_t \\ \begin{bmatrix} x_t \\ y_t \end{bmatrix} &= \begin{bmatrix} b_1 & b_2 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_{t-4} \\ y_{t-4} \end{bmatrix} + \begin{bmatrix} \varepsilon_t \\ 0 \end{bmatrix} \end{aligned}$$



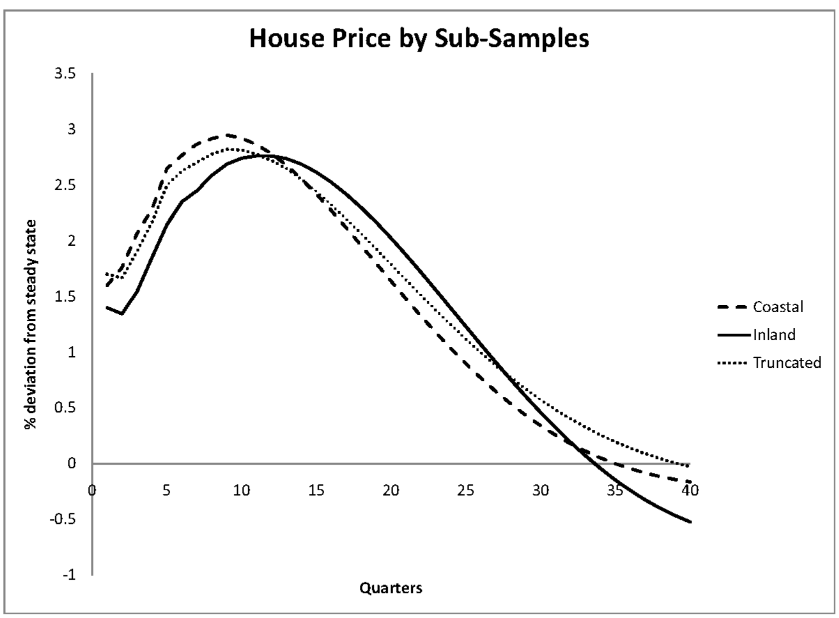


Figure 8: IRFs for quarterly prices

Now consider a VAR(1) given by

$$X_t = \mathbf{A}X_{t-1} + \mathbf{v}_t$$

where $\mathbf{v}_t = [v_t \quad 0]'$. Iterating on this yields

$$X_t = \mathbf{A}^4 X_{t-4} + \mathbf{A}^3\mathbf{v}_{t-3} + \mathbf{A}^2\mathbf{v}_{t-2} + \mathbf{A}\mathbf{v}_{t-1} + \mathbf{v}_t$$

It follows that $\mathbf{A} = \mathbf{B}^{\frac{1}{4}}$ and $\mathbf{e}_t = \mathbf{A}^3\mathbf{v}_{t-3} + \mathbf{A}^2\mathbf{v}_{t-2} + \mathbf{A}\mathbf{v}_{t-1} + \mathbf{v}_t$. We can decompose the VAR(1) as

$$\begin{aligned} x_t &= a_{11}x_{t-1} + a_{12}y_{t-1} + v_t \\ y_t &= a_{21}x_{t-1} + a_{22}y_{t-1} \end{aligned}$$

But since $y_t = x_{t-4}$ this is

$$\begin{aligned} x_t &= a_{11}x_{t-1} + a_{12}x_{t-5} + v_t \\ x_{t-4} &= a_{21}x_{t-1} + a_{22}x_{t-5} \end{aligned}$$

Substituting out x_{t-5} yields

$$\begin{aligned} x_t &= a_{11}x_{t-1} + \frac{a_{12}}{a_{22}}\left(x_{t-4} - a_{21}x_{t-1} - v_{2t}\right) + v_t \\ x_t &= \left(a_{11} - \frac{a_{12}a_{21}}{a_{22}}\right)x_{t-1} + \frac{a_{12}}{a_{22}}x_{t-4} + v_t \end{aligned}$$

Thus the AR(2) process at the annual frequency translates into a particular AR(4) process at the quarterly frequency. There is of course a loss of information.

For incomes the implied AR(4) process is

$$y_t = 1.30y_{t-1} - 0.152y_{t-4} + e_t$$

with $\sigma_e = 0.0091$. Figure 2 shows the impulse response function for a shock such that y_t reaches the same point during the 4th quarter as it does after 1 year for the annual IRF above. The peak occurs after about 2 years.

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